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INFLUENCE OF FREE-STREAM TURBULENCE INTENSITY ON HEAT TRANSFER IN THE TWO-DIMENSIONAL TURBULENT BOUNDARY LAYER OF AN ACCELERATED COMPRESSIBLE FLOW*

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Abstract—For the accelerated compressible flow in a two-dimensional convergent–divergent nozzle the influence of free-stream turbulence on heat transfer in the turbulent boundary layer has been investigated empirically. The turbulence intensity varied from nearly zero to about 20%, the nozzle entrance Reynolds number reached up to about 10^7. The experimental set-up and the turbulence measurement technique are carefully described. For three different loading cases the measured data of free-stream turbulence intensity and fluctuation are given along the nozzle axial length as well as the local Stanton number in comparison to those of accelerated flow without free stream turbulence. For low Reynolds numbers ($Re_x < 10^6$) no clear change of heat transfer has been observed, while for $Re_x > 10^6$ a weak and nearly linear dependency of Stanton number on free-stream turbulence intensity can be pointed out.

NOMENCLATURE

a,	empirical constant;
B,	nozzle width;
c_p,	specific heat at constant pressure;
Ma,	Mach number;
p,	static pressure;
p_t,	total pressure;
$\dot{Q}$,	heat transfer rate;
Re_x,	$\frac{\bar{u} \cdot x}{\nu}$ Reynolds number;
St,	Stanton number;
T,	temperature;
Tu,	free-stream turbulence intensity;
$\bar{u}$,	mean velocity in x-direction;
u', v', w',	velocity fluctuation in x-, y-, z-direction;
Δx,	length of nozzle segment in flow direction;
x, y, z,	Cartesian coordinates: in flow direction; vertical to the segmented wall; parallel to the segmented wall.

Greek symbols

α,	heat transfer coefficient;
ν,	kinematic viscosity;
ρ,	density.

Indices

aw,	adiabatic wall;
eff,	effective;
r,	recovery;
Tu,	with free-stream turbulence.

1. INTRODUCTION

BASED on the knowledge of the influence of strong favourable pressure gradient on heat transfer in a two-dimensional turbulent boundary layer of an accelerated low-turbulence flow from measurements of Winkler *et al.* [1] and Bauer *et al.* [2], the influence of free-stream turbulence intensity on heat transfer in a plane convergent–divergent (CD) nozzle has been investigated empirically. Great interest of aircraft engine industries and only a few measurements of other authors [3, 4], restricted to free-stream turbulence intensities up to about 10% and subsonic flow [14], gave the impulse for our measurements. These measurements enveloped the variation of free-stream turbulence intensity up to about 20% in the whole working range of the existing CD-nozzle with a maximum outlet velocity of about 700 m/s at a free-stream Mach number of 2.2.

2. EXPERIMENTAL SET-UP

The investigations have been performed in the wind tunnel of our institute. Its air circulation is shown in Fig. 1. The air drawn in from the ambient is compressed in a stationary driven two-stage radial compressor (max. pressure ratio $\pi_{max} = 3.4$; max rate of flow $\dot{m}_{max} = 2.8$ kg/s) and by this heated up at maximum to about 485 K. The following three air heaters were not used in the described test series but give the possibility

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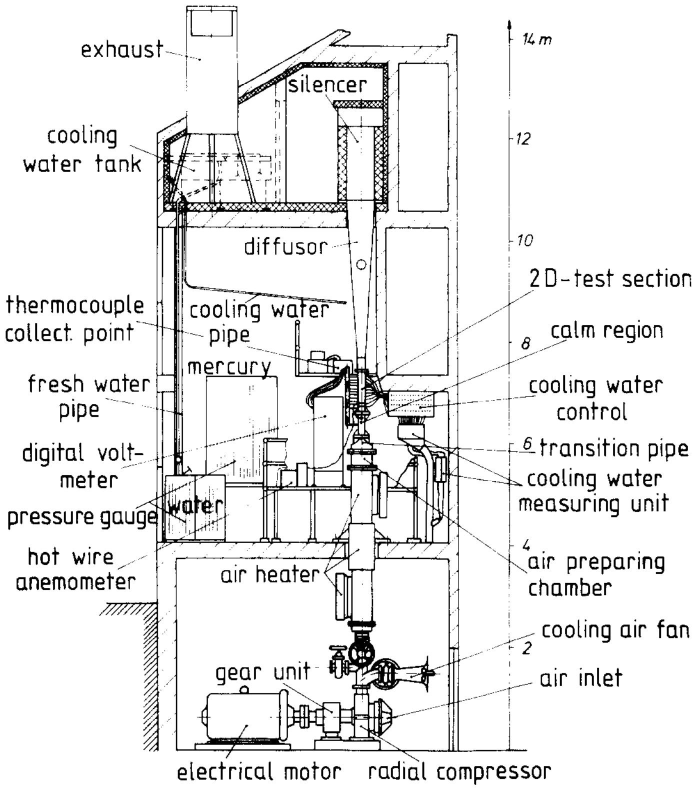

Fig. 1. Plan of the windtunnel facility.

to heat the air additionally. After the air heater the compressed air is prepared in a chamber to smooth the velocity and temperature profile and to install a definite free-stream turbulence intensity. For this purpose a mixer, a straightener and some screens are used, followed by a turbulence grid. The geometric data of the used turbulence grids are given in Table 1. Leaving the preparing chamber the air passes a transition pipe where the cross section is changed from a round to a rectangular one of nearly the same area. Afterwards the heated compressed air enters the two-dimensional test section, an unsymmetrical rectangular water-cooled CD-nozzle (see Section 3) which is designed for a normal shock at its exit in the maximum loading case of the compressor. Passing the diffusor and silencer the hot air leaves the apparatus through an exhaust and returns into the environment. The pressures were measured by U-manometers (mercury or water respectively), from what pictures were taken which can be evaluated later. The voltages of the nearly 250 thermo-couples—necessary for heat flux measurements—were collected by a multi-channel digital voltmeter and punched on paper tape.

Table 1. Geometric data of turbulence grids

Tu-generator no.	Grid bar diameter (mm)	Grid mesh width (mm)
I	3	12.5
II	6	25
III	12	50
IV	24	100

3. TEST SECTION WITH INSTRUMENTATION

The test section has a rectangular cross-section and exists of a plane and a contoured wall (see Figs 2 and 3). To make very accurate measurements of the heat flux from the hot air to the water-cooled nozzle walls the CD-nozzle was built up with 35 separate segments of copper on each of the two side-walls. The segments are well insulated to each other and separately cooled to perform special thermal conditions at the wall, for instance isothermal wall as done in these investigations.

The apparatus is driven in steady state. Therefore heat fluxes into the nozzle walls can be measured by heat flux meters as shown in Fig. 3. For more information about the measurement of static pressure and heat flux along the nozzle see [2] and [5].

Besides the mentioned parameters the stagnation pressure and temperature as well as the free-stream turbulence intensity can be determined at five fixed points in the nozzle-flow (see Fig. 2). A combined temperature and pressure probe as in smaller size used by Meier [6] for boundary layer measurements was employed for measuring the stagnation conditions, while a hot-wire anemometer has been used to determine the change of turbulence intensity in the flow passing through the nozzle.

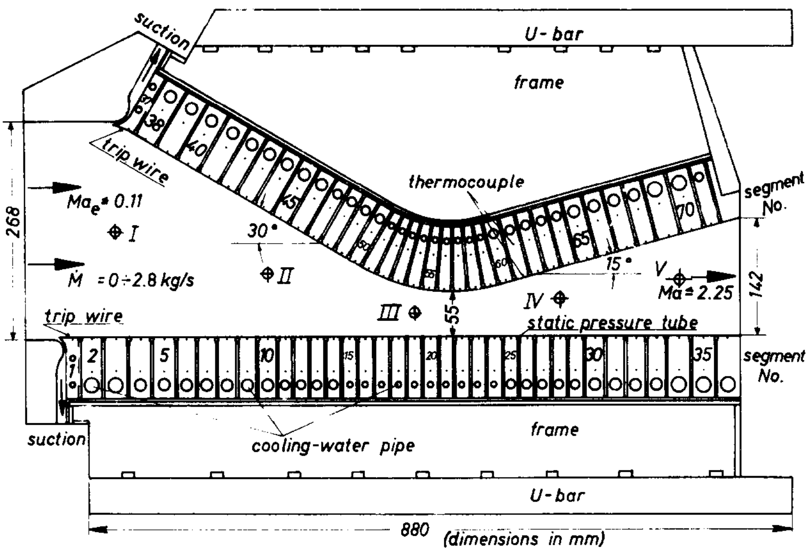

FIG. 2. Longitudinal section of the rectangular convergent–divergent test nozzle. (I to V indicate the location of the hot wire probes.)

4. USED HOT-WIRE PROBE

Although the free-stream flow is one-dimensional the free-stream turbulence is obviously three-dimensional. That means we have to measure the velocity fluctuations in x-, y- and z-axis, i.e. $\sqrt{\overline{u'^2}}$, $\sqrt{\overline{v'^2}}$ and $\sqrt{\overline{w'^2}}$, to determine the turbulence intensity which is defined for the x-direction by the following relation

$$Tu_x = \frac{\sqrt{\overline{u'^2}}}{\bar{u}}. \tag{1}$$

For those measurements we need a so-called X-probe (see Fig. 4a), which exists of two hot wires situated vertically to each other forming a 45°-angle with the main flow direction. Because of the very rough experimental conditions and the expected great amount of failures of the extremely sensitive wire we looked for another probe which is able to do the same job. Acrivlellis [7] has suggested a so-called 45°-probe (see Fig. 4b) which has one wire in a 45°-angle to the main flow direction. Compared to the X-probe the 45°-probe has the advantage to be easier to handle and to repair but the disadvantage not being able to measure all turbulent fluctuations at the same time. So it is not

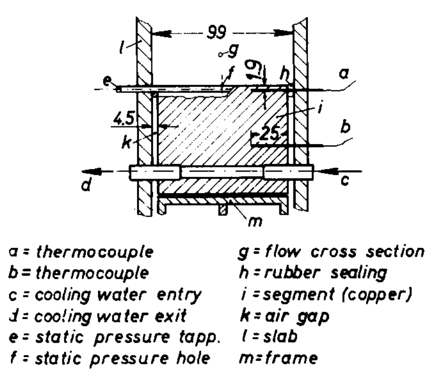

FIG. 3. Cross-section of a segment from the nozzle.

very useful for our measurements. Therefore we decided to take a normal straight hot-wire probe (see Fig. 4c; Type DISA-55 A 75) knowing that this probe can only detect effective velocities and effective turbulent fluctuations. That means we will get an effective turbulence intensity

$$Tu_{x,\mathrm{eff}} = \frac{\sqrt{\overline{u'^2_{\mathrm{eff}}}}}{\bar{u}_{\mathrm{eff}}} \tag{2}$$

where $\overline{u'^2_{\mathrm{eff}}}$ and $\bar{u}_{\mathrm{eff}}$ come from the following relations

$$u_{\mathrm{eff}} = \sqrt{[(\bar{u}+u')^2 + v'^2 + w'^2]} \tag{3}$$

or developed in a series neglecting higher order terms

$$u_{\mathrm{eff}} = \bar{u}\left(1 + \frac{u'}{\bar{u}} + \frac{v'^2 + w'^2}{2\bar{u}^2} - \frac{u'v'^2 + u'w'^2}{2\bar{u}^3} + \dots\right). \tag{4}$$

Using the definition that the mean value per time unit of a turbulent fluctuation term is equal to zero, i.e.

$$\bar{u}' = \int_{t_0}^{t_0+\Delta t} u'\,\mathrm{d}t = 0 \tag{5}$$

we get (see Kristensen [8])

$$\bar{u}_{\mathrm{eff}} = \bar{u}\left(1 + \frac{\overline{v'^2} - \overline{w'^2}}{2\bar{u}^2} - \frac{\overline{u'v'^2} + \overline{u'w'^2}}{2\bar{u}^3}\right). \tag{6}$$

Following the definition

$$u = \bar{u} + u' \tag{7}$$

one can calculate the effective turbulent fluctuation u'_{eff} from equations (4) and (6)

$$u'_{\mathrm{eff}} = \bar{u}\left(\frac{u'}{\bar{u}} + \frac{v'^2 - \overline{v'^2}}{2\bar{u}^2} + \frac{w'^2 - \overline{w'^2}}{2\bar{u}^2} - \frac{u'v'^2 + \overline{u'v'^2}}{2\bar{u}^3} - \frac{u'w'^2 + \overline{u'w'^2}}{2\bar{u}^3}\right) \tag{8}$$

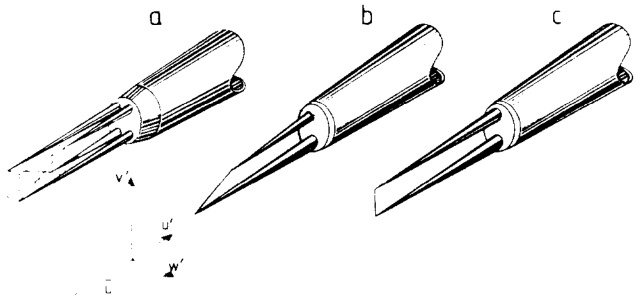

FIG. 4. Available types of hot wire probes; (a) X-probe; (b) 45°-probe; (c) straight probe.

which leads to

$$\overline{u_{\text{eff}}'^2} = \overline{u'^2}\left(1 + \frac{\overline{u'v'^2}}{\bar{u}\cdot\overline{u'^2}} + \frac{\overline{u'w'^2}}{\bar{u}\cdot\overline{u'^2}} + \frac{\overline{v'^4} - (\overline{v'^2})^2}{4\bar{u}^2\cdot\overline{u'^2}} + \frac{\overline{w'^4} - (\overline{w'^2})^2}{4\bar{u}^2\cdot\overline{u'^2}} - \frac{\overline{u'^2v'^2}}{\bar{u}^2\cdot\overline{u'^2}} - \frac{\overline{u'^2w'^2}}{\bar{u}^2\cdot\overline{u'^2}}\right). \quad (9)$$

From equations (4) and (9) one can see that

$$\overline{u_{\text{eff}}} \neq u \quad \text{and} \quad \sqrt{\overline{u_{\text{eff}}'^2}} \neq \sqrt{\overline{u'^2}}.$$

Because we don't like to know u and $\sqrt{\overline{u'^2}}$ but the turbulence intensity Tu, we have to check whether

$$\frac{\sqrt{\overline{u_{\text{eff}}'^2}}}{\bar{u}_{\text{eff}}} \overset{?}{=} \frac{\sqrt{\overline{u'^2}}}{\bar{u}} \quad (10)$$

is guilty or not.

In the case of low turbulent fluctuations, i.e.

$$\frac{u'}{\bar{u}} \ll 1, \quad \frac{v'}{\bar{v}} \ll 1, \quad \frac{w'}{\bar{w}} \ll 1$$

from equations (6) and (9) we get

$$\overline{u_{\text{eff}}} \cong \bar{u} \quad (11a)$$

and

$$\overline{u_{\text{eff}}'^2} \cong \overline{u'^2} \quad (11b)$$

where equation (11a) is more right than equation (11b). That means the measured mean velocity $\bar{u}_{\text{eff}}$ is much less influenced by v' and w' than the turbulent fluctuation $\sqrt{\overline{u_{\text{eff}}'^2}}$. Therefore the turbulence intensity $Tu_{x,\text{eff}}$ of a one-dimensional flow measured by a normal straight hot-wire probe is always a little bit greater then the real existing turbulence intensity in x-direction, Tu_x.

$$Tu_{x,\text{eff}} = \frac{\sqrt{\overline{u_{\text{eff}}'^2}}}{\bar{u}_{\text{eff}}} \geqslant Tu_x = \frac{\sqrt{\overline{u'^2}}}{\bar{u}}.$$

The use of a straight hot-wire probe to measure turbulence intensities of about 20% and more is a strong simplification, indeed. But on one hand hot-wire measurements are erroneously in the order of 20% in general and on the other hand the decay of turbulence in the nozzle is very rapid and therefore the error decreases too. Another point is that we would like to know the behaviour and change of turbulence along the nozzle and its influence on heat transfer to the cooled wall, and not the real value of turbulence intensity extremely accurate. Therefore we decided to use a straight high-temperature hot-wire probe with the advantages of a simple structure, an easy repair and a relative low effort for measurements. The measured effective turbulence intensity $Tu_{x,\text{eff}}$ should be used as the representative turbulence intensity to characterize the behaviour of the turbulent free-stream along the CD test nozzle. To simplify the notation let us introduce Tu instead of $Tu_{x,\text{eff}}$.

5. TEST RESULTS

The working range of the test nozzle is given by the measured pressure ratio p_x/p_t and the free-stream Mach number Ma_x vs the axial length of the nozzle in Figs. 5 and 6. For the investigation described in this paper the loading cases 3, 7 and 12 have been selected, that means minimum, medium and maximum load. While the heated air is accelerated throughout the nozzle for maximum load (loading case 12), it is accelerated for medium load (7) up to the shock in the divergent part of the nozzle. For minimum load (3) the flow is accelerated till the nozzle throat and decelerated afterwards. In this paper heat transfer behaviour in the boundary layer under the influence of free-stream turbulence should not be looked at for flow conditions over and behind the shock as well as decelerated flow, i.e. the investigation is restricted to the favourable pressure gradient flow only.

The decay of the turbulence intensity Tu and of the turbulent fluctuation $\sqrt{\overline{u'^2}}$ in the nozzle is plotted in Figs. 7 and 8 for minimum load and in Figs. 9 and 10 for medium or maximum load respectively. One can see that Tu drops very rapidly for all loading cases, perhaps a little bit stronger for medium and maximum load, for what no turbulence measurements have been successfully in the throat region and at the nozzle exit because of the very rough flow condition (high temperature and/or high velocity), which the hot-wire could not bear.

In all cases the turbulence intensity is nearly down to the basic turbulence of the arrangement just behind the throat. The decay of Tu is not mainly based on the

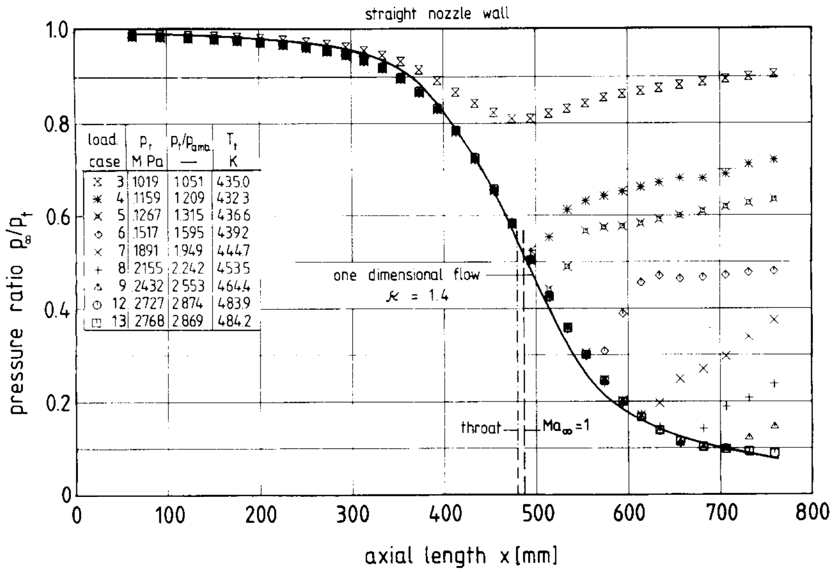

FIG. 5. Static-to-total pressure ratio p_∞/p_t vs axial length x.

increase of the mean free-stream velocity $\bar{u}$ because the turbulent fluctuation $\sqrt{\overline{u'^2}}$ decays in a comparable way as Tu does.

Looking at the influence of free-stream turbulence on heat transfer to the cooled wall one will observe a very small influence in general. Therefore the heat transfer results are given in Figs. 11–13 in relation to the heat transfer of the favourable pressure gradient flow without free-stream turbulence, that means with exactly the basic turbulence of the arrangement only.

The heat transfer is expressed by the Stanton number St defined as follows

$$St = \frac{\alpha_r}{\rho_\infty \cdot u_\infty \cdot c_p} \tag{12}$$

where the heat transfer coefficient α_r comes from

$$\alpha_r = \frac{\dot{Q}/B}{(T_{aw} - T_w) \cdot \Delta x}. \tag{13}$$

For minimum load (Fig. 11) with a Reynolds number of about $Re_x \cong 10^5$ at the nozzle entrance nearly no influence of free-stream turbulence decay on heat transfer can be seen up to an axial length of x = 300 mm. Only the heat transfer data for the different turbulence grids and the therefore different turbulence intensity levels deviate from each other, nearly parallel shifted. Near the throat and in the divergent part of the nozzle a slight increase of heat transfer is observed while the turbulence intensity decreases. The decelerated flow should not be discussed in this paper.

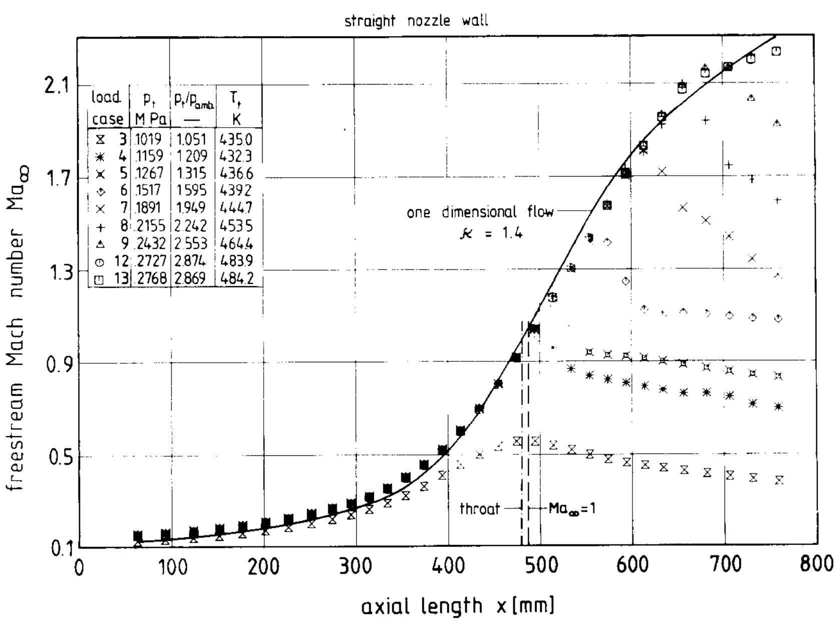

FIG. 6. Free-stream Mach number Ma_∞ vs axial length x.

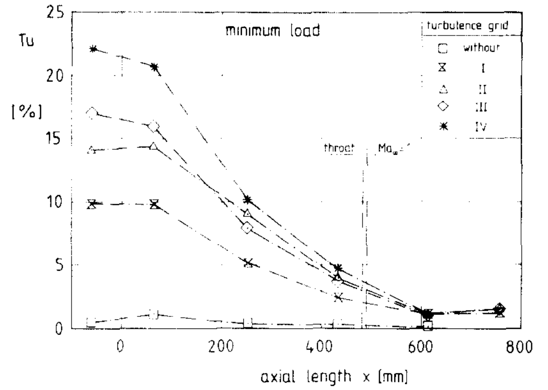

FIG. 7. Free-stream turbulence intensity Tu vs axial length x for minimum load.

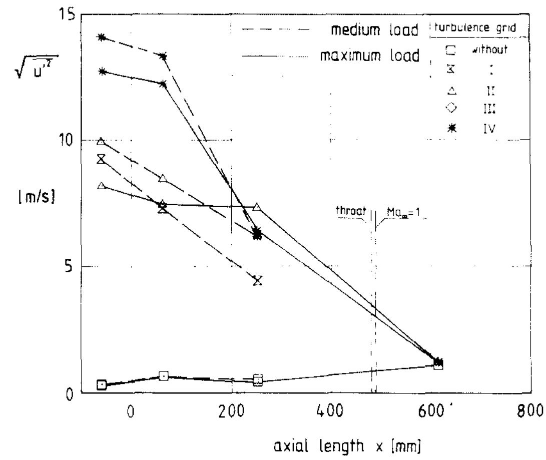

FIG. 10. Free-stream turbulent fluctuation $\sqrt{u'^2}$ vs axial length x for medium and maximum load.

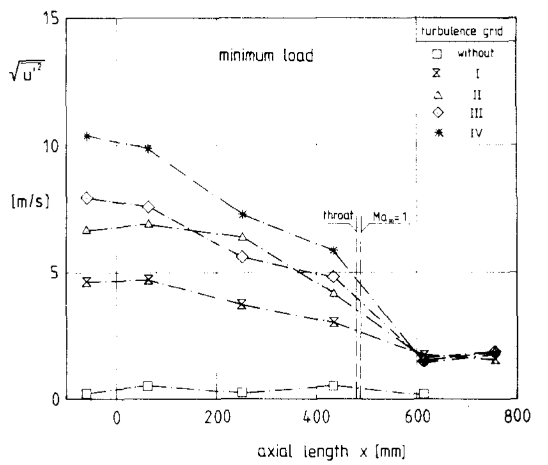

FIG. 8. Free-stream turbulent fluctuation $\sqrt{u'^2}$ vs axial length x for minimum load.

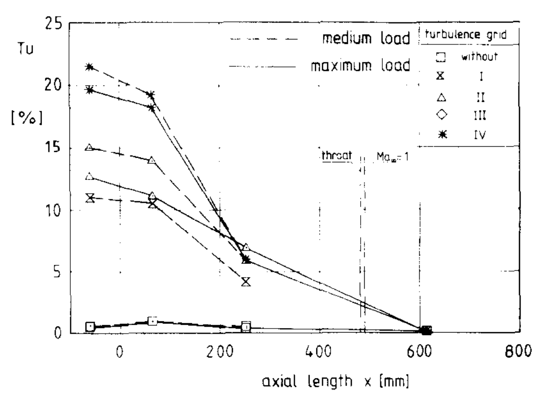

FIG. 9. Free-stream turbulence intensity Tu vs axial length x for medium and maximum load.

The small increase in front of the throat can be explained by the increasing Reynolds number into a range greater than $Re_x = 10^6$ which seemed to be an important barrier for the influence of free-stream turbulence on heat transfer in the compressible turbulent boundary layer. This assumption is emphasized by the results of the own measurements for medium and maximum load (see Figs. 12 and 13), what for the Reynolds number from the beginning of the nozzle is $Re_x \geqslant 10^6$. For both loading conditions and influence of free-stream turbulence intensity on heat transfer is very clear to be seen. The change of the Stanton number for existing free-stream turbulence St_{Tu} compared to the Stanton number without free-stream turbulence St_0, i.e.

$$\frac{St_{Tu} - St_0}{St_0} = f(x)$$

shows along the axial nozzle length nearly the same slope in decay as the free-stream turbulence intensity Tu itself, what can be seen by the additionally given Tu-curves (dotted line in Figs. 12 and 13) for the turbulence grids number II and IV (see Table 1). The drawn curves illustrate the optical average of the Stanton numbers coming directly from the measurement. A correction of the measured data has been abandoned because the effect on the Stanton number is in the same order as the error of measurements (see especially in Fig. 13); but in any case the described tendency can be seen obviously.

From our own measurements the following empirical relation can be derived for $Re_x \geqslant 10^6$ to describe the dependency of heat transfer on free-stream turbulence intensity

$$St_{Tu} = St_0\,(1 + a \cdot Tu)$$

with (14)

$$a \cong 0.7 \div 1.25 \cong 1.$$

Concluding our own results one can say that the influence of free-stream turbulence on heat transfer in

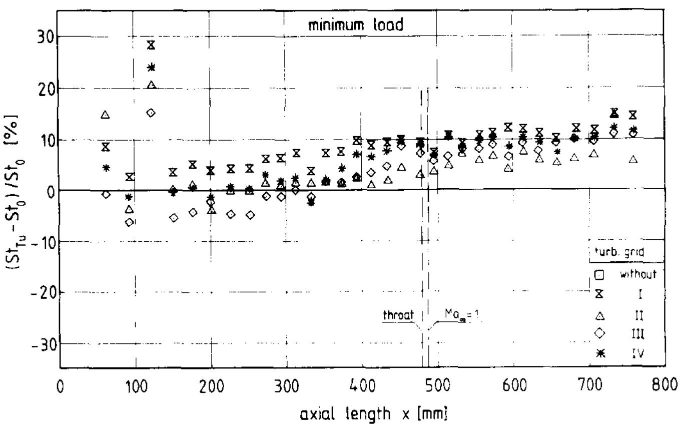

FIG. 11. Stanton number for turbulent free-stream St_{T_u} in comparison to Stanton number for non-turbulent free-stream St_0 vs axial length x for minimum load.

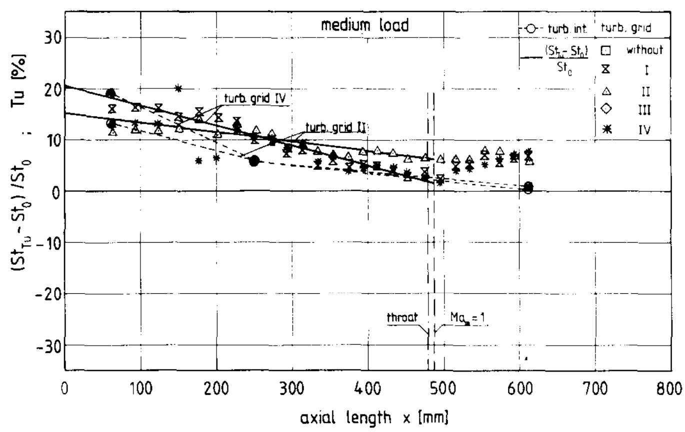

FIG. 12. Stanton number for turbulent free-stream St_{T_u} in comparison to Stanton number for non-turbulent free-stream St_0 vs axial length x for medium load.

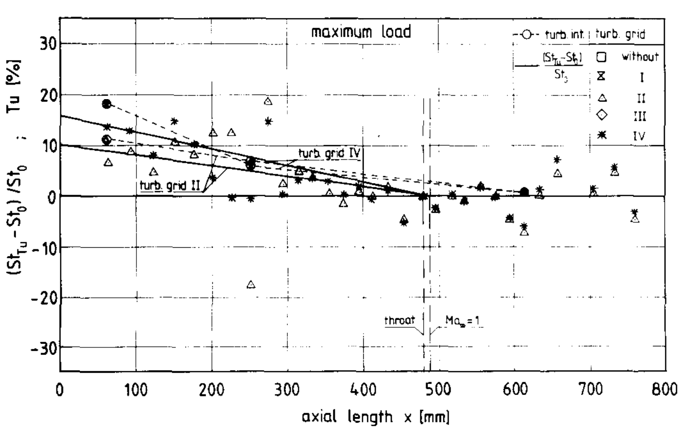

FIG. 13. Stanton number for turbulent free-stream St_{T_u} in comparison to Stanton number for non-turbulent free-stream St_0 vs axial length x for maximum load.

a two-dimensional turbulent boundary layer of an accelerated compressible flow is

(a) in agreement with the results of Simonich and Bradshaw [9] and the suggestion of McDonald and Kreskovsky [10] uneffective for Reynolds numbers lower than $Re_x \cong 10^6$ and

(b) for $Re_x \geqslant 10^6$ also for free-stream turbulence intensities up to 20% really low as already measured for lower turbulence intensities by Boldman [3] and Brown and Burnton [4]; that means in our case, the heat transfer increases nearly proportional to the increase of free-stream turbulence intensity.

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INFLUENCE DE L'INTENSITE DE TURBULENCE DE L'ECOULEMENT LIBRE SUR LE TRANSFERT THERMIQUE DANS LA COUCHE LIMITE TURBULENTE BIDIMENSIONNELLE D'UN ECOULEMENT COMPRESSIBLE ACCELERE

Résumé—On étudie pour un écoulement compressible accéléré à l'intérieur d'une tuyère convergente-divergente, l'influence de la turbulence de l'écoulement libre sur le transfert thermique. L'intensité de turbulence varie depuis zéro jusqu'à 20% environ, le nombre de Reynolds à l'entrée de la tuyère allant jusqu'à 10^7 environ. Le montage expérimental et la technique de mesure de la turbulence sont décrits en détail. Pour trois charges différentes, les résultats sur l'intensité de turbulence et la fluctuation sont donnés le long de l'axe de la tuyère et le nombre de Stanton local est comparé au cas de l'écoulement accéléré sans turbulence libre. Pour les faibles nombres de Reynolds ($Re_x < 10^6$), on n'observe pas de changement net du transfert thermique, tandis que pour $Re_x \geqslant 10^6$ on peut noter une dépendance faible et presque linéaire du nombre de Stanton vis-à-vis de l'intensité de turbulence libre.

EINFLUß DER FREISTROMTURBULENZ AUF DEN WÄRMEÜBERGANG IN DER ZWEIDIMENSIONALEN TURBULENTEN GRENZSCHICHT EINER BESCHLEUNIGTEN KOMPRESSIBLEN STRÖMUNG

Zusammenfassung—Für die beschleunigte kompressible Strömung in einer zweidimensionalen konvergent-divergenten Düse wurde der Einfluß der Freistromturbulenz auf den Wärmeübergang in der turbulenten Grenzschicht experimentell untersucht. Der Turbulenzgrad wurde zwischen 0 und 20% variiert, die Reynoldszahl am Düseneintritt betrug bis zu 10^7. Die Versuchsanlage und die Turbulenz-Meßtechnik werden ausführlich beschrieben. Für drei verschiedene Lastfälle werden die Meßdaten der Freistromturbulenz und turbulenten Schwankungsgröße entlang der Düse als auch der lokalen Stantonzahlen im Vergleich zu denen einer beschleunigten Strömung ohne Freistromturbulenz angegeben. Für kleine Reynoldszahlen ($Re_x < 10^6$) war keine deutliche Änderung des Wärmeübergangs zu beobachten, während für $Re_x \geq 10^6$ eine schwache, etwa lineare Abhängigkeit der Stantonzahl von der Freistromturbulenz herausgearbeitet werden konnte.

ВЛИЯНИЕ ИНТЕНСИВНОСТИ ТУРБУЛЕНТНОСТИ СВОБОДНОГО ПОТОКА НА ТЕПЛОПЕРЕНОС В ДВУХМЕРНОМ ТУРБУЛЕНТНОМ ПОГРАНИЧНОМ СЛОЕ ПРИ УСКОРЕННОМ ТЕЧЕНИИ СЖИМАЕМОЙ ЖИДКОСТИ

Аннотация — Проведено эмпирическое исследование влияния турбулентности основного потока на теплоперенос в турбулентном пограничном слое при течении сжимаемой жидкости в двухмерном сопле Лаваля. Интенсивность турбулентности изменялась в диапазоне от нуля до ~20%, значение числа Рейнольдса на входе в сопло составляло примерно 10^7. Дано подробное описание экспериментальной установки и методики измерений турбулентности. Для трех режимов течения измерены значения интенсивности турбулентности основного потока вдоль оси сопла, а также определено локальное значение числа Стэнтона и дано сравнение со случаем ускоренного течения, в котором отсутствует турбулентность основного потока. При малых значениях числа Рейнольдса ($Re_x < 10^6$) не было отмечено явных изменений в картине теплопереноса, в то время как при $Re_x \geq 10^6$ наблюдалась слабая и почти линейная зависимость числа Стэнтона от интенсивности турбулентности свободного потока.